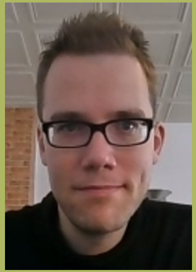
CMCS 2026

Turin

Locally Final Coalgebras in Denotational Semantics



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April 2026

Contents

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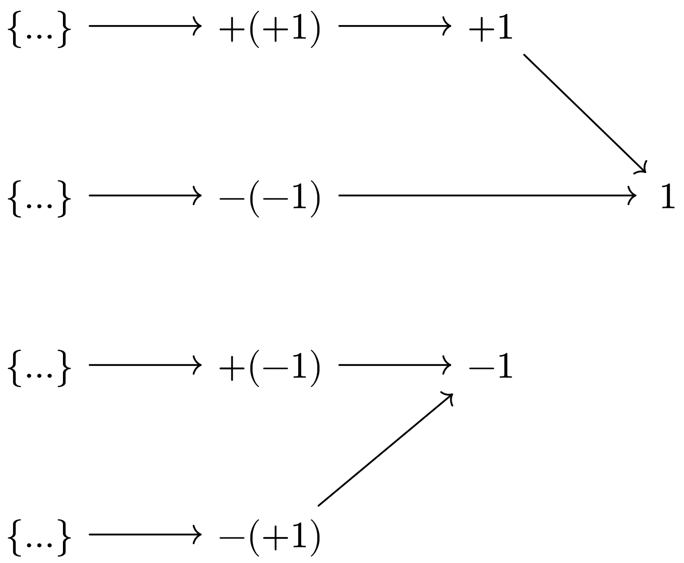
Denotational Semantics

Two Traditions

Operational Semantics

(Some flavour of) transition system! The set of nodes is $\mathcal{L}$, the set of all programs.



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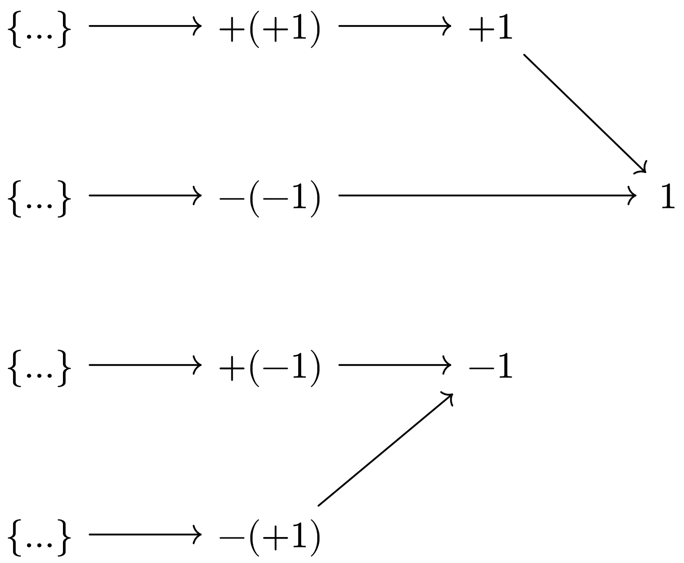

Denotational Semantics

The way a logician would do it (compositionally, hopefully). A recursive definition inducing a map

$$\begin{aligned} [\![-]\!] &: \mathcal{L} \to \mathbb{B} \\ [\![1]\!] &:= \mathrm{true} \\ [\![+n]\!] &:= [\![n]\!] \\ [\![-n]\!] &:= \neg [\![n]\!]. \end{aligned}$$

(Not) Reusing Theory in PL

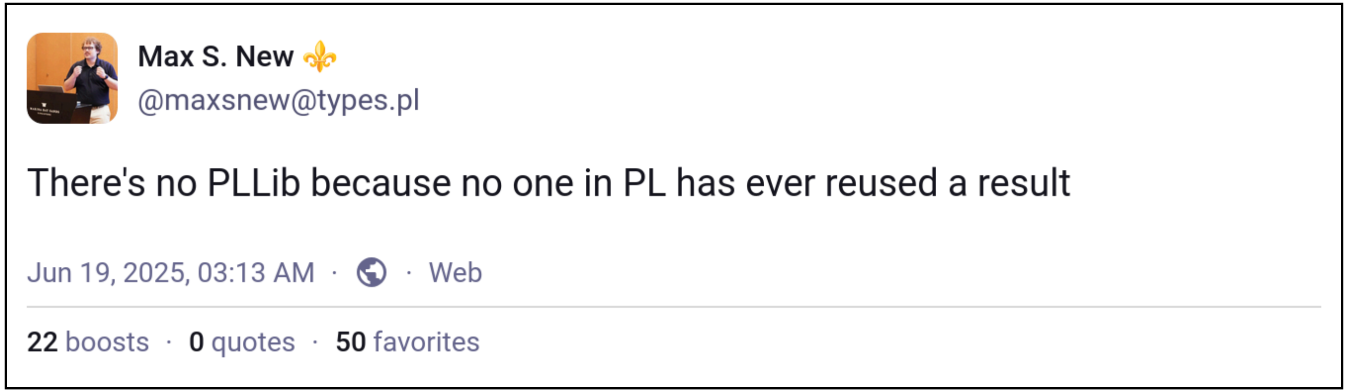

Figure 1: [1] [Online]. Available: https://types.pl/@maxsnew/114707374331592652

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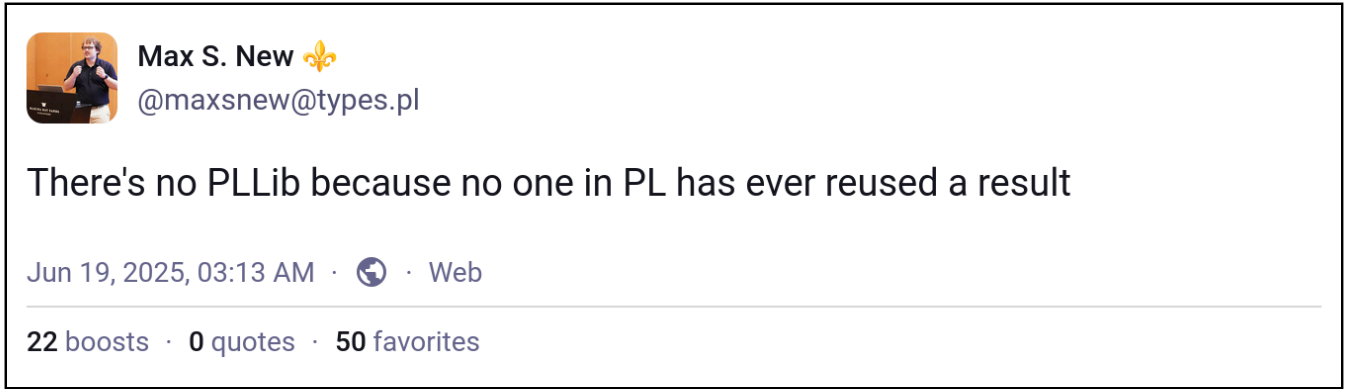

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Reuse is one of the benefits of **abstraction**.

~~Operational~~ Coalgebraic and ~~Denotational~~ Algebraic Semantics

Operational Semantics

as a **coalgebra** over a behaviour endofunctor $B : \mathscr{C} \to \mathscr{C}$
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Also, if we coinductively defined $Z := \nu B$, it carries a **final coalgebra** structure ζ.

$$\Sigma(\nu B) \overset{a}{\to} \nu B \overset{\zeta\cong}{\to} B(\nu B)$$

What's the pattern?

Abstract GSOS (Grand Structured Op. Sem.)

Assume $\mathcal{C}$ has binary products, Σ has an algebraically free monad $\Sigma^\star$, and that we encoded the derivation system for the operational model as a natural transformation

$$\begin{array}{cc} \dfrac{\cdots \quad \cdots}{\cdots} & \dfrac{\cdots}{\cdots} \\[1em] \dfrac{\cdots \quad \cdots \quad \cdots}{\cdots} & \dfrac{\cdots \quad \cdots}{\cdots} \end{array} \quad \rightsquigarrow \quad (\rho_X : \Sigma(X \times BX) \to B\Sigma^\star X)_{X \in \mathrm{Ob}(\mathcal{C})}.$$

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Then, ρ-*bialgebras* are pairs (g, h) of a Σ-coalgebra g and a B-coalgebra h with a shared carrier making

$$\begin{array}{ccccc} \Sigma X & \xrightarrow{\quad g \quad} & X & \xrightarrow{\quad h \quad} & BX \\ \Big\downarrow{\scriptstyle \Sigma\langle \mathrm{id}_X, h\rangle} & & & & \Big\downarrow{\scriptstyle B\hat{g}} \\ \Sigma(X \times BX) & & \xrightarrow{\qquad \rho_X \qquad} & & B\Sigma^\star X \end{array}$$

commute. We take these as models: algebra-coalgebra pairs respecting the derivation system.

Abstract GSOS (Grand Structured Op. Sem.) (2)

ρ-bialgebras form a category, where morphisms are maps which are algebra and coalgebra morphisms at the same time.

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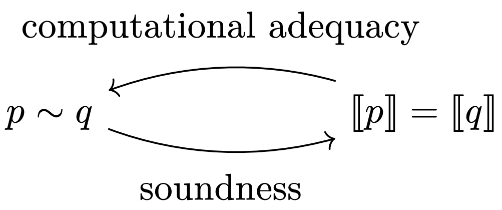
D. Turi and G. Plotkin [2] proved the existence of an **initial** and of a **final object**, and that they are the **operational** and **denotational models** respectively.

Both are compositional. Computational adequacy holds. If B preserves weak pullbacks, so does soundness.



$$\text{full abstraction} = \text{soundness} + \text{computational adequacy}$$

Higher-order Abstract GSOS… (1)

Higher-order behaviour requires a mixed-variance behaviour functor, $B : \mathcal{C}^{\mathrm{op}} \times \mathcal{C} \to \mathcal{C}$ (e.g., $B(X, Y) = Y + Y^X$ in a category with binary products and exponentials).

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Assume $\mathcal{C}$ has binary products, Σ has an initial algebra $\mu\Sigma \to \Sigma(\mu\Sigma)$, as well of an algebraically free monad $\Sigma^\star$, and that we have

$$\left(\rho_{\boldsymbol{X},Y} : \Sigma(X \times B(\boldsymbol{X}, Y)) \to B(\boldsymbol{X}, \Sigma^\star(\boldsymbol{X} + Y))\right)_{\boldsymbol{X},Y \in \mathrm{ob}(C)}$$

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dinatural in X and natural in Y. Now, ρ-bialgebras are easily adapted [3], while a morphism is now a map $f : X \to Y$ s.t.

$$\begin{array}{ccccccc}
\Sigma X & \xrightarrow{g_X} & X & \xrightarrow{h_X} & B(X,X) & & \\
 & & & & & \searrow^{B(X,f)} & \\
\downarrow{\scriptstyle \Sigma f} & & \downarrow{\scriptstyle f} & & & & B(X,Y) \\
 & & & & & \nearrow_{B(f,X)} & \\
\Sigma Y & \xrightarrow{g_Y} & Y & \xrightarrow{h_Y} & B(Y,Y) & &
\end{array}$$

commutes.

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Operational…

S. Goncharov, S. Milius, L. Schröder, S. Tsampas, and H. Urbat [3] proved the existence of an **initial ρ-bialgebra**, and that it is a **compositional operational model**.

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…But Not Denotational (yet)

A final (diagonal) higher-order coalgebra does not always exist, nor it always extends to a final ρ-bialgebra…

…Now with Denotational Semantics!

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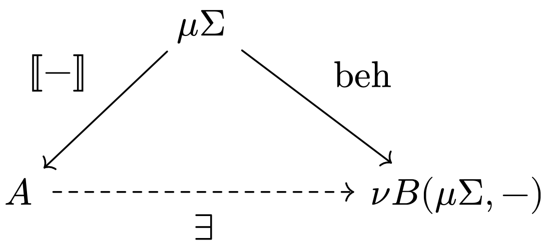

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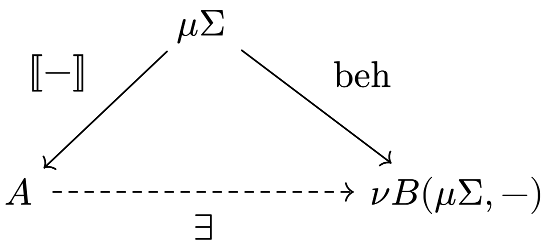

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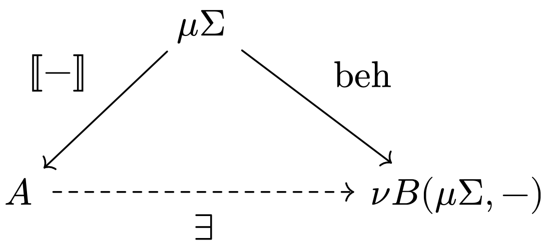

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A map $Z \to B(Z, Z)$ that is a final $B(Z, -)$-coalgebra is called a *locally final coalgebra*. If ρ is relatively flat, every locally final coalgebra extends to a ρ-bialgebra! How to find a locally final coalebra?

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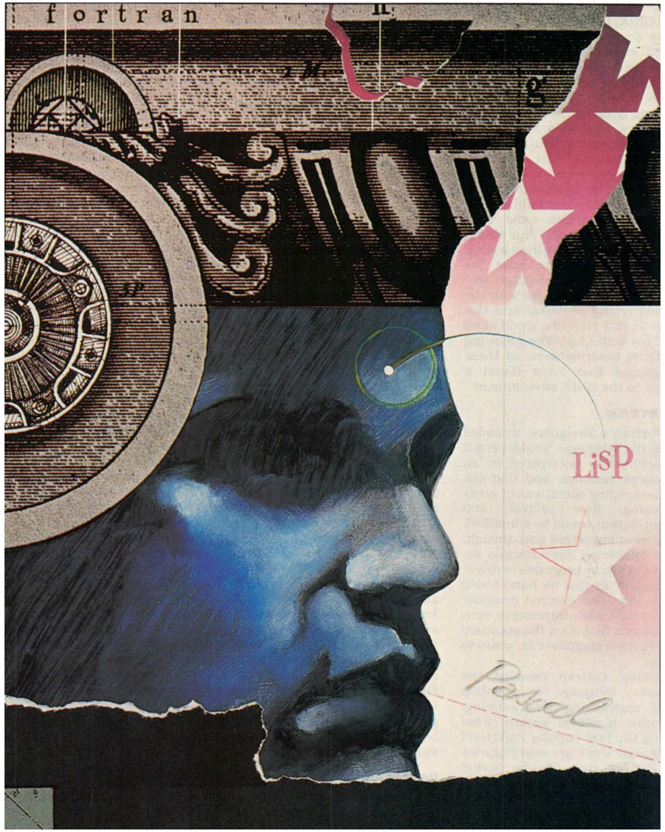

Figure 3: [4] Unknown artist, *UNIX REVIEW cover*. 1985.

Locally Final Coalgebras

A Metric Setting for Fixpoints

The Setting

By Lambek, locally final coalgebras are particular fixpoints of B seen as a recursive equation[1]. Can we adapt previous fixpoint theory?

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Assume $\mathcal{C}$ to be an M-category, i.e. a category enriched over complete, bounded metric spaces satisfying the ultrametric equality:

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(Slight rephrasing of) L. Birkedal, K. Støvring, and J. Thamsborg [5]:

If $\mathcal{C}$ has a terminal object and inverse limits of increasing Cauchy towers, every locally contractive functor F such that there exists a map $1 \to F1$ has a fixpoint.

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If all hom-sets are inhabited, the fixpoint is unique.

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Iterative Construction

Adámek's Final Sequence

If $\mathcal{C}$ has a terminal object and limits of chains (i.e., α^{op}-chains for all ordinals α), we can start from 1 and repeatedly apply a fixed $H : \mathcal{C} \to \mathcal{C}$ to construct the *final sequence*, a (large) diagram $D : \mathrm{Ord}^{\mathrm{op}} \to \mathcal{C}$.

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Stable M-Category

M-category $\mathcal{C}$ is stable if, for every parallel pair $f, g : A \to B$ and jointly monic family $(h_i : B \to C_i)_{i \in I}$,

$$d_{A,B}(f, g) = \sup_{i \in I} d_{A,C_i}(h_i \circ f, h_i \circ g)$$

($\geq$ always holds by non-expansivity of $\circ$).

Existence and Uniqueness of Locally Final Coalgebras

Let $\mathcal{C}$ be a stable M-category with a terminal object and limits of chains. Let $B : \mathcal{C}^{\mathrm{op}} \times \mathcal{C} \to \mathcal{C}$ be such that:

- B is locally contractive in the first argument, locally non-expansive in the second;
- there is a map $1 \to B(1, 1)$;
- for each $X \in \mathrm{ob}(\mathcal{C})$, the final sequence of $B(X, -)$ converges.

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Proof Idea

Convergence for each X yields a contravariant functor

$$F : \mathcal{C}^{\mathrm{op}} \to \mathcal{C}$$
$$F(X) := \nu B(X, -).$$

We are looking for a fixpoint of F, so we can use the fixpoint theorem for both existence and uniqueness.

Contents

Figure 4: [6] Cooksey-Talbott Studio, “Beyond PCs: The Unix System Integrates the Office.” 1985.

Conclusions

Summary

Issue

How can we easily derive denotational semantics from operational semantics of higher-order languages without reinventing the wheel?

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How can we easily derive denotational semantics from operational semantics of higher-order languages without reinventing the wheel?

Contributions

- → A construction of a computationally adequate, compositional **categorical model from any locally final coalgebra** (assuming relative flatness only).
- → A theory of **existence and uniqueness of locally final coalgebras** via (ultra)metric enrichments.

Future Work

Big-Step Operational Semantics

Our ρ can only encode small-step rules (HO-GSOS), so we get strong applicative bisimilarity. What about big-step rules (AHOS)? We could get *weak* applicative bisimilarity.

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Other Enrichment-Based Fixpoint Theories

→ Enrichment over CPOs (domain theory!)

→ Enrichment over sheaves over a fixed, complete Heyting algebra with a well-founded basis (transfinite step-indexing!)

Takeaway

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Locally final coalgebras make them easily derivable from operational semantics!

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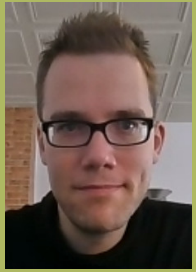

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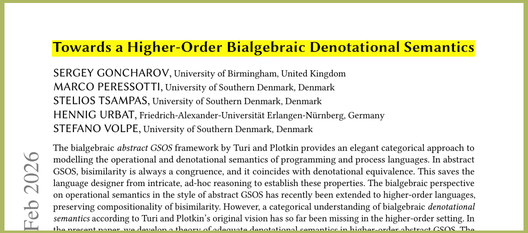
arxiv.org/abs/2602.18295



Towards a Higher-Order Bialgebraic Denotational Semantics

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MARCO PERESSOTTI, University of Southern Denmark, Denmark
STELIOS TSAMPAS, University of Southern Denmark, Denmark
HENNIG URBAT, Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany
STEFANO VOLPE, University of Southern Denmark, Denmark

Feb 2026

The bialgebraic *abstract GSOS* framework by Turi and Plotkin provides an elegant categorical approach to modelling the operational and denotational semantics of programming and process languages. In abstract GSOS, bisimilarity is always a congruence, and it coincides with denotational equivalence. This saves the language designer from intricate, ad-hoc reasoning to establish these properties. The bialgebraic perspective on operational semantics in the style of abstract GSOS has recently been extended to higher-order languages, preserving compositionality of bisimilarity. However, a categorical understanding of bialgebraic *denotational semantics* according to Turi and Plotkin's original vision has so far been missing in the higher-order setting. In the present paper, we develop a theory of adequate denotational semantics in higher-order abstract GSOS. The

Thank you!

References

[1] [Online]. Available: https://types.pl/@maxsnew/114707374331592652

[2] D. Turi and G. Plotkin, “Towards a mathematical operational semantics,” in *Twelfth Annual IEEE Symposium on Logic in Computer Science*, IEEE Comput. Soc, 1997, pp. 280–291. doi: 10.1109/lics.1997.614955.

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[7] S. Goncharov, M. Peressotti, S. Tsampas, H. Urbat, and S. Volpe, “Towards a Higher-Order Bialgebraic Denotational Semantics,” 2026. [Online]. Available: https://arxiv.org/abs/2602.18295

Relative Flatness

We assume a mild condition on ρ which, in derivation systems, has the following interpretation:

Constructors from signature Σ can be well-ordered so that, if $f \in \Sigma$ is assigned rank j, term $f(...)$ transitions into terms of strictly lower terms, except for their head symbol, which may have rank $\leq j$.

Idea: ban builtin Y combinator and whatnot.

Relating the Two Semantics

Logic

Syntax and semantics:

completeness: $\Gamma \vdash \varphi \leftarrow \Gamma \models \varphi$

soundness: $\Gamma \vdash \varphi \rightarrow \Gamma \models \varphi$

$$\text{adequacy} = \text{soundness} + \text{completeness}$$

Relating the Two Semantics

Logic

Syntax and semantics:

$$\Gamma \vdash \varphi \;\underset{\text{soundness}}{\overset{\text{completeness}}{\leftrightarrows}}\; \Gamma \models \varphi$$

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Programming Languages (PL)

“Syntactic” semantics (with fixed behavioural equivalence $\sim$) and “traditional” semantics:

$$p \sim q \;\underset{\text{soundness}}{\overset{\text{computational adequacy}}{\leftrightarrows}}\; [\![p]\!] = [\![q]\!]$$

$$\text{full abstraction} = \text{soundness} + \text{computational adequacy}$$